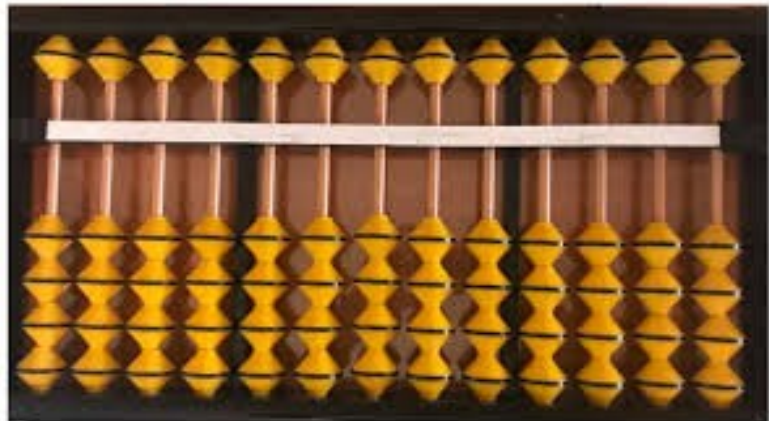


Computer Chips Demystified

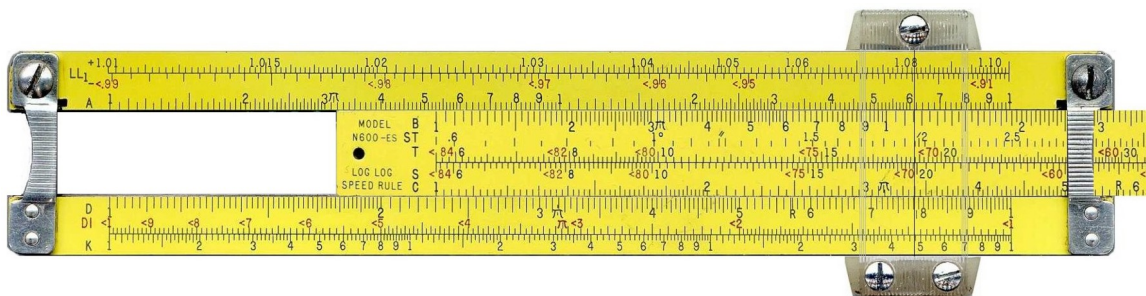
History

Humankind has always had their 10 fingers for basic math. However when counting above 10, humans have been using machines to help perform basic arithmetic long before modern civilization. As you will see now, advanced computers can perform little more than basic arithmetic, they can just do it on a very grand scale. Far before ancient civilization in 35000 BC, the Lebombo bone was created in southern Africa. It is a baboon fibula with 29 distinct notches (ref [1]). Archaeologists believe this linear counter is the earliest known mathematical tool used by humans.

The Abacus: The abacus was utilized in Babylonia as early as c. 2700 – 2300 BC. A form of abacus uses 5 beads per wire with each adjacent wire representing a power of 10 increase. The single bead above the bar represents a value of 5. The 4 beads below the bar represent incremental values. Sliding a bead above the bar upward represents adding 5 to the digit; sliding a bead upward below the bar represents incrementing the digit value by one (ref [2]).



Slide Rules & Logarithm Tables: The slide rule was introduced in the 1620's. A slide rule is a sliding logarithmic scale between two fixed logarithmic scales. Its application is in multiplication / division / exponents (ref [3]). This “go to” device was the principal tool of Scientists & Engineers until the introduction of the electronic pocket calculator in the 1970's. The math applications of multiplication, division & exponents could be implemented on slide rules with application of incremental logarithmic values calculated by John Napier (1550 – 1617) (ref [4]) & introduced @ about the same time. In general, calculations using slide rules & log tables were accurate to about 3 significant digits.



A logarithm is the reverse of an exponential. Two logarithm bases are important, base 10 & base “e”. Take a calculus course to find out the mysteries of base “e”, the so-called

natural logarithm. Base 10 logarithm & reverse logarithm tables are used for the following calculations (ref [5]).

$$y = 10^x \Leftrightarrow \log_{10}(y) \equiv x$$

For multiplication using Napier's logarithm tables,

$$y_1 \cdot y_2 = 10^{x_1} \cdot 10^{x_2} = 10^{x_1+x_2}$$

$$\log_{10}(y_1)|_{\text{table}} + \log_{10}(y_2)|_{\text{table}} = x_3 \Leftrightarrow y_1 \cdot y_2 = 10^{x_3}|_{\text{table}}$$

For division using Napier's logarithm tables,

$$y_1 / y_2 = 10^{x_1} / 10^{x_2} = 10^{x_1} \cdot 10^{-x_2} = 10^{x_1-x_2}$$

$$\log_{10}(y_1)|_{\text{table}} - \log_{10}(y_2)|_{\text{table}} = x_3 \Leftrightarrow y_1 / y_2 = 10^{x_3}|_{\text{table}}$$

For exponentials using Napier's logarithm tables,

$$y^{1/2} = (10^{x_1})^{1/2} = 10^{0.5 \cdot x_1}$$

$$0.5 \cdot |\log_{10}(x_1)|_{\text{table}} = x_3 \Leftrightarrow y^{1/2} = 10^{x_3}|_{\text{table}}$$

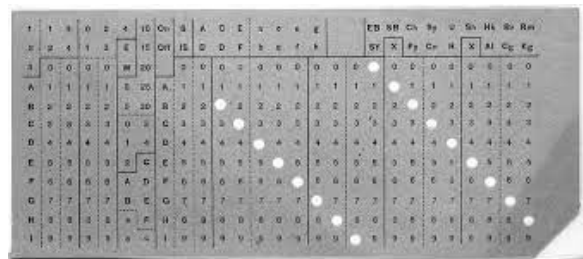
Note: Pocket calculators supersede slide rules & the abacus for any important mathematical calculations, in modern times.

Mechanical Adding Machines: In 1642, Blaise Pascal (1623 – 1662) (ref [6]) invented a working mechanical adding machine for his father, a tax collector. These & following mechanical adding machines put the linear slide rule on a rotating wheel. A value was set by placement on the wheel.

In 1672, Gottfried Leibniz (refs [7] & [8]) introduced a stepped drum adding machine. More importantly, Leibniz refined the binary system in applying mathematics & is considered the "founder of Computer Science". If all of these achievements weren't enough for Leibniz's resumé, he independently invented Calculus along with [Isaac Newton](#) (1642 – 1726) (ref [9]). It is Leibniz's "fractional" notation for derivatives (dg/dt) that is generally used more than Newton's "dot" notation (\dot{g}) in Calculus (ref [10]). BTW, Herr Leibniz is German; the "ei" has an "EYE" sound, "Libe•nitz" (ref [11]).



Punch Cards: In 1804, Joseph Marie Jacquard (1752 – 1834) (ref [12]) proposed the Jacquard loom that helped implement various weave designs into textiles as they were being woven. Punch cards contained a given weave pattern. Subsequent cards determined when to change the weave & to what extent. In the late 1880's, Herman



Hollerith (1860 – 1929) (ref [13]) applied Jacquard's punch cards to data storage. The 1890 US census data tabulation was reduced by 2 years down to 6 years due to this automation. In typical punch card applications, the card only has one human-readable

line, usually an 80 character line @ the top. The card's remainder is dedicated for the machine communication interface.

Alan Turing (1912 – 1954) developed an abstract machine that solved computer algorithms & he is considered the “father of artificial intelligence” (ref [14]). Turing proposed the “Turing machine” that was able to solve by algorithm any programming task if given a near infinite amount of computer memory. Practically all modern programming languages meet this requirement.

The Software of One's & Zero's

Computers store data through the standards of text strings, integers, floating point. Of course, the programmer is not bound to these standards (ref [15]).

The Language of a Computer Chip: All a computer knows is 1's & 0's. All a computer can calculate is addition, subtraction, multiplication & division. The term “byte” was coined by an early International Business Machine (IBM) researcher in 1956 to address a group of 8 bits. A bit is a single binary value, either a one or zero. A group of eight bits is a “byte” deliberately misspelled to *not* be confused with a “bit” (ref [16]).

Multi-Bit Representation of Integers									
#bits	hi value	0	1	2	3	4	5	6	7
1	$2^1-1=1$	0=0	1=1						
2	$2^2-1=3$	00=0	01=1	10=2	11=3				
3	$2^3-1=7$	000=0	001=1	010=2	011=3	100=4	101=5	110=6	111=7
4	$2^4-1=15$	0000=0	0001=1	0010=2	0011=3	0100=4	0101=5	0110=6	0111=7
		1000=8	1001=9	1010=10	1011=11	1100=12	1101=13	1110=14	1111=15

For an 8-bit number, the far right bit is the “least significant bit” & holds a 0 or 1 value multiplied by 2^0 or unity. The far left bit is the “most significant bit” & stores a 1 or 0 value multiplied by 2^7 or 128. The bits in between help refine the span of the 8-bit value from 0 to 2^8-1 or 0 to 255.

“Bit fiddling” may not be required by the programmer with today's GigaBytes of memory. However, if one wants to get into the 1's & 0's of programming, most default calculators on laptop operating systems (Windows / Apple / Linux) have a “Programmer” setting for decimal / binary / octal / hexadecimal evaluations & bit-wise shift functions (ref [17]).

Text Storage: Seven bits allow an American Standard Code for Information Interchange (ASCII pronounced “ASS-key”) letter to be represented (ref [18]). This idea immediately reveals the “secret” behind data storage in a computer. Data can **only** be stored as 1’s & 0’s. It’s how the computer software decides to interpret the 1’s & 0’s that’s the “key”. Likewise, if one doesn’t have the “key”, the 1’s & 0’s are meaningless.

The following chart list printable ASCII characters (32 through 127) of the 128 available values. Decimal (base 10) as well as octal (base 8) & hexadecimal (base 16) values are also listed. Octal uses 8 digits (0-7) to represent an integer. For easier decoding, each octal digit corresponds to 3 bytes of data. Hexadecimal uses 16 digits (0-9, A-F) to represent an integer. Each hexadecimal value represents 4 bytes of binary data. Note, a blank space (SP) is assigned the decimal number 32.

Unicode is the descendant of ASCII & represents the world’s myriad language alphabets as 161 different scripts & emoji’s through 2 adjacent bytes or 16 bits (ref [19]). Two adjacent bytes can represent 256 alphabet scripts of 256 letters. A programmer “strings” adjacent bytes or byte pairs together & begins to “write” a sentence. For the fonts, all UNICODE letters have a “free” default font, but the user can also designate his / her own.

ASCII Printable Characters											
decimal	octal	hexadecimal	text	decimal	octal	hexadecimal	text	decimal	octal	hexadecimal	text
32	40	28	SP	64	100	64	@	96	140	8C	.
33	41	29	!	65	101	65	A	97	141	8D	a
34	42	2A	"	66	102	66	B	98	142	8E	b
35	43	2B	#	67	103	67	C	99	143	8F	c
36	44	2C	\$	68	104	68	D	100	144	90	d
37	45	2D	%	69	105	69	E	101	145	91	e
38	46	2E	&	70	106	6A	F	102	146	92	f
39	47	2F	'	71	107	6B	G	103	147	93	g
40	50	32	(72	110	6E	H	104	150	96	h
41	51	33)	73	111	6F	I	105	151	97	i
42	52	34	*	74	112	70	J	106	152	98	j
43	53	35	+	75	113	71	K	107	153	99	k
44	54	36	,	76	114	72	L	108	154	9A	l
45	55	37	-	77	115	73	M	109	155	9B	m
46	56	38	.	78	116	74	N	110	156	9C	n
47	57	39	/	79	117	75	O	111	157	9D	o
48	60	3C	0	80	120	78	P	112	160	A0	p
49	61	3D	1	81	121	79	Q	113	161	A1	q
50	62	3E	2	82	122	7A	R	114	162	A2	r
51	63	3F	3	83	123	7B	S	115	163	A3	s
52	64	40	4	84	124	7C	T	116	164	A4	t
53	65	41	5	85	125	7D	U	117	165	A5	u
54	66	42	6	86	126	7E	V	118	166	A6	v
55	67	43	7	87	127	7F	W	119	167	A7	w
56	70	46	8	88	130	82	X	120	170	AA	x
57	71	47	9	89	131	83	Y	121	171	AB	y
58	72	48	:	90	132	84	Z	122	172	AC	z
59	73	49	;	91	133	85	[123	173	AD	{
60	74	4A	<	92	134	86	\	124	174	AE	
61	75	4B	=	93	135	87]	125	175	AF	}
62	76	4C	>	94	136	88	^	126	176	B0	~
63	77	4D	?	95	137	89	_	127	177	B1	DEL

Integer Storage: A mathematician defines an integer as a whole number 0, 1, 2, 3, 4 increasing to infinity plus the negative of whole numbers (ref [20]). Computers can accommodate a mathematician’s definition of non-negative integers with an upper limit. Positive numbers can be represented well enough with 4 bytes or 32 bits. The “unsigned” variable can range from 0 to (2³²-1) or 0 to 4294967295 or 0 to about 4 billion. Negative integers are represented in various ways but essentially a single bit is reserved for setting to a positive or negative flag. Then, a “signed” integer can range from (1-2³¹) to (2³¹-1) or -2147483647 to 2147483647 or about -2 billion to +2 billion.

Floating Point Storage: A floating point representation uses scientific notation to represent a range of values from the very small to the very large both positive & negative. Floating point typically uses a 64-bit or 128-bit data structure where the upper section of bits is dedicated to the “exponent” & the remaining lower bits hold the “mantissa” value. For -3.3344×10^{-2} , -3.3344 is the mantissa or significand & -2 is its exponent.

Supporting such a structure requires much bit manipulation. For example, to add two floating point values, the exponents must be made equivalent by shifting mantissas right or left, the mantissas are added & the combined exponent is then adjusted. With current technology, this is all hardwired in a “co-processor” to be performed as fast as possible for floating point structures when supported.

IEEE 754 Floating Point Structure							
official	name common	# bits	# bytes	mantissa	exponent	value limits	
						underflow	overflow
binary32	half float	32	4	24	8	6.0E-5	6.0E+4
binary64	float	64	8	53	11	1.1E-38	1.1E+38
binary128	double	128	16	113	15	2.2E-308	2.2E+308
binary256	long double	256	32	237	19	3.3E-4932	3.3E+4932

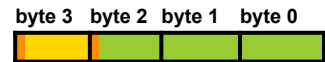
Floating point formats & accuracies are defined by standards developed by the Institute of Electrical and Electronics Engineers (IEEE pronounced “EYE triple EE”) (ref [21]). For programming languages, when an American National Standards Institute (ANSI pronounced “AN-see”) version of a compiler is specified, like FORTRAN-66 (developed in 1966) or FORTRAN-77 (developed in 1977), equivalent floating point accuracy & structure are specified & inherent in the compiler (ref [22]). The floating point requirements are carried through & implemented in the produced code. The above / below charts identify common parameters of IEEE 754 floating point data (refs [23] & [24])

The charts identify “half float” & “long double” that are seldom used. For example, in floating calculations of Einstein’s [Special Relativity](#), the activity often involves manipulating values close to the magnitude of light speed (about 10^9 kph) with significant bits in the lowest significant area. Answers are many times unattainable with less than “double” precision.

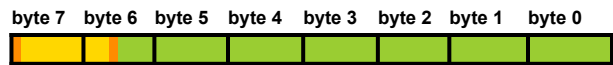
For division by zero & other errors, the floating point format can also assume IEEE pre-assigned values such as NaN (not a number) (ref [25]) or “arithmetic overflow” in mathematical calculation errors.

IEEE 754 Floating Point Formats

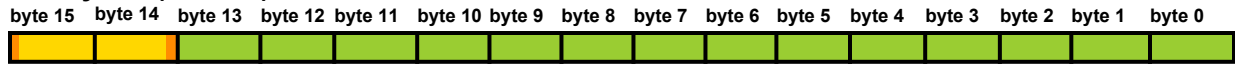
binary32 (half float)



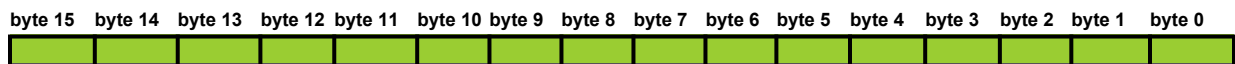
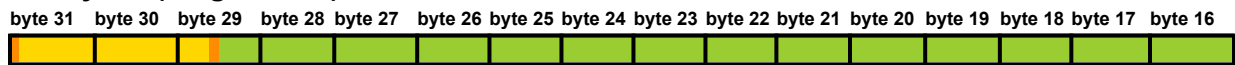
binary64 (float)



binary128 (double)



binary256 (long double)



Boolean Storage: Paired truth values (true / yes or false / no) were initially defined by George Boole (1815 – 1864) for truth logic evaluations (ref [26]). In modern times, the variable is used in programming for ON / OFF flags or conditional evaluations. A conditional answer for storage is the results of an example question: “is $x < y$?” The boolean value needs a single bit size, but the variable usually carries a 16-bit or 32-bit size. In program execution, if the variable has any value other than zero, the stored evaluation is true, otherwise it’s false. Mister Boole was an Englishman & his name “BOOL” rhymes with “POOL” (ref [27]).

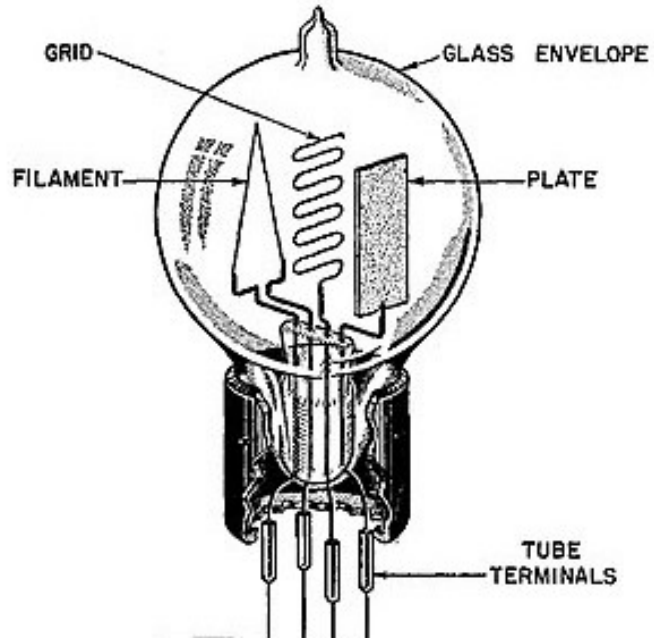
Computer Chip Machinery

The 1st part of this article covered the **great** flexibility of binary “1” or “0” data storage. With the proper format, a binary virtual world can approximate the real world to any extent given enough computer memory. Now let’s look @ where this virtual world physically resides on a microchip.

The Vacuum Tube: The vacuum tube was invented by John Ambrose Fleming in 1904 (ref [28]). Its invention is the forerunner of the transistor that makes our digital world possible.

The simplest vacuum tube function is the diode. A diode allows electric current to flow in one direction only. A diode only has 2 terminals, a cathode (filament) & anode (plate). With a voltage drop applied to a diode, electrons flow from its cathode to its anode in a vacuum. Adding one or more control grids to this early diode allows the “load current” flow to be modified based on a smaller “input current” applied to the control “grid”. With the control “grid”, the vacuum tube becomes an amplifier (ref [29]). BTW, if the electrons from the cathode impinge on an anode covered in light-emitting phosphorus, one has a Cathode Ray Tube (CRT), the bulky, heavy glass pre-plasma television of past ubiquity (refs [30] & [31]).

The transmission of EM signals carrying information was due in part to the efforts of a distinguished list of Physics researchers of the 19th century, including [Michael Faraday](#) (1791 – 1867) (ref [32]), [James Maxwell](#) (1831 – 1879) (ref [33]), [Nikola Tesla](#) (1856 – 1943) (ref [34]) & Guglielmo Marconi (1874 – 1937) (ref [35]). Once generated, the EM signals are received @ remote locations where amplification of the detected signals is required. For the most part, an emitted signal strength decreases with an inverse distance squared per Coulomb's Law. In the 1920's, @ sea ship-to-ship communications & music / news from land-based radio stations both required the function of a vacuum tube for remote radio signal amplification (ref [36]).



The Transistor: To replace the vacuum tube, the silicon-based field-effect transistor was demonstrated in 1947 by Physicists John Bardeen, Walter Brattain & William Shockley while working @ AT&T Bell Labs in New Jersey (ref [37]). “The most widely used transistor” type is the metal-oxide-semiconductor field-effect transistor (MOSFET), invented at Bell Labs in 1959 (ref [38]).

Several types of silicon-based transistors exist, but MOSFET's are “the workhorse of the modern electronics industry” (ref [39]) & will be examined further here. Vacuum tubes are a brute force method of manipulating electric current using metal conductors & high energy electrons streaming in a partial vacuum. On the other hand, low voltage electrons are passing through a silicon semiconductor crystal lattice in a MOSFET. The scale of transistors verses vacuum tubes are magnitudes smaller resulting in less energy loss & more efficiency in computation applications.

A typical MOSFET circuit transistor (ref [40]) replaces the vacuum tube in both form & function. To operate properly, all vacuum tube components take place in a significant albeit partial vacuum. In a transistor, the principle medium is an almost pure silicon lattice.



Silicate rocks in nature give one the feel of almost pure silicon of computer chips. Quartz is tough, durable, extremely difficult to fracture. These qualities are evidence of the strong bonds within the Quartz's SiO₂ (Silicon-Oxygen) lattice. Practically, all available electrons form covalent bonds with adjacent lattice atoms. Quartz crystals have similar strength to pure Silicon: Mohs hardness is measured @ 7 & 6.5 for Quartz

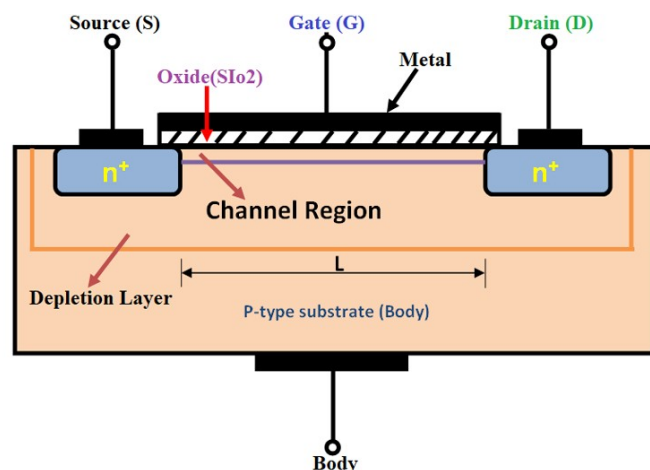
& pure Silicon, respectively; their melting points are similar @ 3030°F & 2580°F for Quartz & pure Silicon, respectively (refs [41] & [42]).

Doping: A pure Silicon ($_{14}\text{Si}$) structure has a relatively high electrical resistivity of about 1000 $\text{n}\Omega\cdot\text{m}$ verses 17 $\text{n}\Omega\cdot\text{m}$ for Copper @ room temperature (ref [42]). A process called “doping” is used to place single atoms of different elements in the $_{14}\text{Si}$ lattice of a transistor to give the semiconductor any degree of electric conductance, as needed ... hence, the meaning of the name “semiconductor”. The table below gives a partial list of doping elements used in chip manufacture. The element Silicon has a 14-electron configuration $[_{10}\text{Ne}] 3s^2 3p^2$. Here, the element Neon is abbreviated Ne with 10 orbiting electrons. In this strange shell notation, “s” & “p” are electron (e) subshells with 2 electrons each for $_{14}\text{Si}$ ($14 = 10+2+2$). The notation is derived from spherical harmonics & the Periodic Table discussed later.

name	sym	electron configuration	uses
Positive Doping Elements			
Boron	$_5\text{B}$	$[_2\text{He}] 2s^2 2p^1$	common in CMOS uses
Gallium	$_{31}\text{Ga}$	$[_{18}\text{Ar}] 3d^{10} 4s^2 4p^1$	used in long-infrared sensors
Indium	$_{49}\text{In}$	$[_{36}\text{Kr}] 4d^{10} 5s^2 5p^1$	used in mid-infrared sensors
Negative Doping Elements			
Phosphorus	$_{15}\text{P}$	$[_{10}\text{Ne}] 3s^2 3p^3$	neutron capture of silicon
Arsenic	$_{33}\text{As}$	$[_{18}\text{Ar}] 3d^{10} 4s^2 4p^3$	equals silicon atomic diameter
Antimony	$_{51}\text{Sb}$	$[_{36}\text{Kr}] 4d^{10} 5s^2 5p^3$	used in VLSI circuits

In a $_{14}\text{Si}$ lattice, $_{14}\text{Si}$ atoms form 4 covalent bonds with 4 adjacent $_{14}\text{Si}$ atoms. N-doping elements have 5 electrons; p-doping elements have 3 electrons. P-doping elements need one electron to complete their characteristic “stable” subshells & have a missing electron hole, in effect, a positive p-doping. N-doping elements have one free negative electron after filling their characteristic “stable” subshells.

Vacuum Tube vs. Transistor: In a vacuum tube, the filament (cathode) ejects energized, mobile electrons as a high voltage electron stream. In the transistor element, a metallic conductor supplies elevated voltage electrons to a doped region of the $_{14}\text{Si}$ lattice termed a “source”. The “grid” of a vacuum tube that accepts the current to amplify, is termed the “gate” in a transistor. The vacuum tube plate (anode) that receives the modulated electron stream has a similar area in a transistor lattice termed the “drain”. In addition, to aid in manufacturing, the transistor is fabricated on a doped silicon base called the “body”. The transistor “body” is essentially @ an overall ground Voltage for the transistor (ref [43]).



The Gate Capacitor: Besides doping that increases the current conductance of a ${}_{14}\text{Si}$ lattice, an insulator silicate region (SiO_2 – silicon-dioxide) is employed. Here, a SiO_2 layer is deposited before the metallic coating of the gate, forming a capacitor.

A capacitor is an insulated break in a circuit where opposite charges build up on either side when a voltage drop is applied (ref [44]). A capacitor is a crude battery in effect, but the capacitor discharges its stored charges in an exponential decay. A battery discharges its stored energy @ a nearly constant voltage. Under the SiO_2 layer is the doped body ${}_{14}\text{Si}$ lattice of the transistor. A weak signal voltage to amplify is applied to the “gate”, the region below the SiO_2 insulator attracts or repels charges to modulate the current flowing from “source” to “drain”.

Let’s answer the earlier question posed, “Where does the virtual world of binary ‘1s’ & ‘0s’ reside on a semiconductor chip?” The answer is: “The virtual world is in the electron charges (or lack thereof) @ the ‘gate capacitor’ of each transistor that stores its binary state.”

Periodic Table of the Elements

The periodic table is color-coded by groups: 1A (blue), 2A (purple), 3A (green), 4A (cyan), 5A (blue), 6A (orange), 7A (red), 8A (yellow), 3B (pink), 4B (light blue), 5B (light green), 6B (light blue), 7B (light green), 8 (light blue), 9 (light green), 10 (light blue), 11B (light green), 12B (light blue), 13A (orange), 14A (green), 15A (blue), 16A (orange), 17A (red), 18A (yellow), 1 (blue), 2 (purple), 13 (orange), 14 (green), 15 (blue), 16 (orange), 17 (red), 18 (yellow), 3 (pink), 4 (light blue), 5 (light green), 6 (light blue), 7 (light green), 8 (light blue), 9 (light green), 10 (light blue), 11 (light green), 12 (light blue), 13 (orange), 14 (green), 15 (blue), 16 (orange), 17 (red), 18 (yellow), 19 (pink), 20 (purple), 21 (light blue), 22 (light green), 23 (light blue), 24 (light green), 25 (light blue), 26 (light green), 27 (light blue), 28 (light green), 29 (light blue), 30 (light green), 31 (orange), 32 (green), 33 (blue), 34 (orange), 35 (red), 36 (yellow), 37 (pink), 38 (purple), 39 (light blue), 40 (light green), 41 (light blue), 42 (light green), 43 (light blue), 44 (light green), 45 (light blue), 46 (light green), 47 (light blue), 48 (light green), 49 (orange), 50 (green), 51 (blue), 52 (orange), 53 (red), 54 (yellow), 55 (pink), 56 (purple), 57-71 (light blue), 72 (light green), 73 (light blue), 74 (light green), 75 (light blue), 76 (light green), 77 (light blue), 78 (light green), 79 (light blue), 80 (light green), 81 (orange), 82 (red), 83 (purple), 84 (light blue), 85 (light green), 86 (light blue), 87 (pink), 88 (purple), 89-103 (light blue), 104 (light green), 105 (light blue), 106 (light green), 107 (light blue), 108 (light green), 109 (light blue), 110 (light green), 111 (light blue), 112 (light green), 113 (orange), 114 (red), 115 (purple), 116 (light blue), 117 (light green), 118 (light blue), 119 (pink), 120 (purple), 121 (light blue), 122 (light green), 123 (light blue), 124 (light green), 125 (light blue), 126 (light green), 127 (light 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The Semiconductor Materials



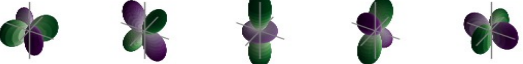
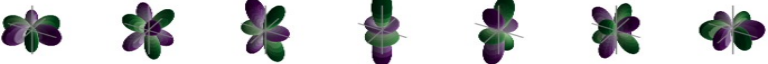
The Periodic Table: In producing solid-state circuits for computers, various elements are used in their pure atomic states. All elements have electrons. The question is: “How free are an element’s electrons?” To a large extent, the Periodic Table (ref [45]) answers that question & to some degree determines which elements to use in a solid-state circuit. In the 19th century, several proposals about the form of a Periodic Table of Elements were made. In 1869, Dmitri Mendeleev (1834 –1907) proposed the chemical periodic table in much of its present form, based on known element properties (ref [46]).

Atomic Nucleus: In 1911, Ernest Rutherford (1871 – 1937) (ref [47]) described

directing high-energy “alpha” particles into different metal foils & measuring their deflection angle. Alpha particles are Helium (${}_2\text{He}$) nuclei stripped of their 2 orbiting electrons, having a (2+) positive charge & denoted (α^{2+}) or (He^{2+}). These α^{2+} -particles also carry 2 neutrons (${}^4\text{He}$). Rutherford generated α^{2+} -particles from the radioactive decay of Thorium (${}_{90}\text{Th}$), weakly radioactive as compared to Uranium (${}_{92}\text{U}$).

By observing α^{2+} -particle reflection angles of 180° from Platinum (${}_{78}\text{Pt}$) foil, he noted it is “as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.” From these experiments, Rutherford proposed the nuclear atomic model where the electron orbits establish an atomic radius around an extremely dense proton/neutron nucleus whose radius is about (now measured @) 100,000 smaller. (ref [48])

Spherical Harmonics: In the 1920’s, Niels Bohr (1885 –1962) & others adapted spherical harmonics to the filling of electron shells & subshells around Rutherford’s nuclear atom (ref [49]) to reflect Mendeleev’s chart. Electrons fill subshells around an atomic nucleus as dictated by spherical harmonic basis functions ($Y_\ell^m(\theta,\varphi)$) (ref [50]). Here, (θ) is the polar angle in relation to an externally applied electromagnetic (EM) field; (φ) is the azimuthal angle.

Electron Subshell Orbits through Spherical Harmonics	
quantum orientation	$m = 0$
quantum angular momentum $\ell = 0$ $1s^2, 2s^2, 3s^2, 4s^2, 5s^2, 6s^2, 7s^2$	
quantum orientation	$m = -1$ $m = 0$ $m = 1$
quantum angular momentum $\ell = 1$ $2p^6, 3p^6, 4p^6, 5p^6, 6p^6, 7p^6$	
quantum orientation	$m = -2$ $m = -1$ $m = 0$ $m = 1$ $m = 2$
quantum angular momentum $\ell = 2$ $3d^{10}, 4d^{10}, 5d^{10}, 6d^{10}$	
quantum orientation	$m = -3$ $m = -2$ $m = -1$ $m = 0$ $m = 1$ $m = 2$ $m = 3$
quantum angular momentum $\ell = 3$ $4f^{14}, 5f^{14}$	

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At a zero index of ($\ell = 0, m = 0$), an s subshell has no angular momentum around the nucleus. As electrons are given increasing angular momentum, ($\ell = 1, 2, 3$), their subshell orbits are allowed various 3D orientations ($\ell = 1, m = -1, 0, +1$) or ($\ell = 2, m = -2, -1, 0, +1, +2$) such that $(-\ell \leq m \leq +\ell)$. ($Y_\ell^m(\theta,\varphi)$) basis functions correspond to s, p, d & f, with more “lobes” and “toruses” (donuts) developing. The above illustration plots spherical harmonic subshells for s, p, d, f with different orientation states of (m) for

increasing angular momentum (ref [51]). In each case, the electron can pass through its atomic nucleus in a process called “tunneling”. The functions of spherical harmonics form a basis set. This requirement dictates that basis functions satisfy a unique function, the Kronecker Delta function.

Kronecker Delta function: This odd function helps explain why the Spherical Harmonic functions are termed an “orthonormal basis set”. Along with a basis set is defined an “inner product”. The term “ortho” reflects that these functions “seem” orthogonal or perpendicular as defined by the “inner product” operation. The term “normal” indicates an inner product of a specific function with itself gives a unity value (ref [52]).

$$\iint Y_{\ell 1}^{m1}(\theta, \varphi) Y_{\ell 2}^{m2}(\theta, \varphi)^* \sin\theta \, d\theta \, d\varphi = \delta_{\ell 1, \ell 2} \delta_{m1, m2} \quad (\text{inner product definition})$$

with solid angle (Ω) integration limits

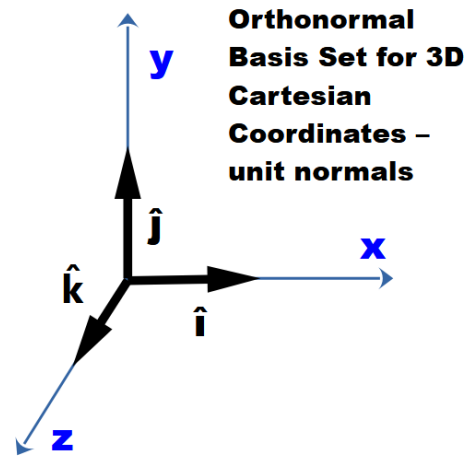
$$\begin{aligned} \varphi: 0 &\rightarrow 2\pi & (0^\circ &\rightarrow 360^\circ) \\ \theta: 0 &\rightarrow \pi & (0^\circ &\rightarrow 180^\circ) \end{aligned}$$

This function (δ_{ab}) is defined as (ref [53]):

$$\begin{aligned} a = b &\quad \Leftrightarrow \quad \delta_{ab} \equiv 1 \\ a \neq b &\quad \Leftrightarrow \quad \delta_{ab} \equiv 0 \end{aligned}$$

Recall in 3D Cartesian space, 3 perpendicular normalized vectors ($\hat{i}, \hat{j}, \hat{k}$) where

$$\begin{aligned} \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} &= \cos(90^\circ) = \delta_{ab} = 0 \\ \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} &= \cos(0^\circ) = \delta_{aa} = 1 \end{aligned}$$



($\hat{i}, \hat{j}, \hat{k}$) form a orthonormal basis set. The vector dot product sets the inner product.

Basis Set Purpose: Spherical harmonic applications are applied best in spherical symmetric conditions. The Coulomb inverse-radius central potential from a positively charged atomic nucleus is just such a condition. Then, spherical angle variables (θ, φ) are adequate with a separate range function $R(r)$ such that $R(r) \cdot f(\theta, \varphi)$ expresses the function form to express through the basis set. Now, ask a mathematician to supply a set of discrete functions that when summed together with set coefficient constants ($C_{\ell m}$), could evaluate any 3D field outside of an atom. Oddly, the mathematician would choose the orthonormal basis set ($Y_{\ell}^m(\theta, \varphi)$) spherical harmonics (ref [54]) as described by Pierre-Simon Laplace (1749 – 1827) in 1782. Then,

$$C_{\ell m} \equiv \iint f(\theta, \varphi) \cdot Y_{\ell}^m(\theta, \varphi)^* \sin\theta \, d\theta \, d\varphi \quad \Leftrightarrow \quad f(\theta, \varphi) = \sum_{\ell} \sum_{m} C_{\ell m} \cdot Y_{\ell}^m(\theta, \varphi)$$

To test this usage, choose $f(\theta, \varphi) = Y_p^q(\theta, \varphi)$ as a 3D field to approximate. Then,

$$\delta_{\ell p} \delta_{mq} = C_{\ell m} = \iint Y_p^q(\theta, \varphi) \cdot Y_{\ell}^m(\theta, \varphi)^* \sin\theta \, d\theta \, d\varphi$$

$$f(\theta, \varphi) = \sum_{\ell} \sum_{m} \delta_{\ell p} \delta_{mq} \cdot Y_{\ell}^m(\theta, \varphi) = Y_p^q(\theta, \varphi) \quad (\text{the initial function has been recovered})$$

Quantum Mechanics (QM) is bizarre! Electron orbits are “quantized” about the central force of a positive atomic nucleus based on the math of a modified Spherical Harmonics basis set. Not only that, the fundamentals of this basis set were devised by a

mathematician over a century before the set was QM applied! The correspondence is not one-to-one, though. The (3d¹⁰ – 6d¹⁰) d subshells & (4f¹⁴ – 5f¹⁴) f subshells are not filled in their correct order. This divergence occurs supposedly because inner electron orbits shield outer electrons from the full central atomic potential.

The Fermion & Its Spin: An electron reacts in some experiments as a point particle, yet it carries a negative charge that can only rotate or “spin” @ one quantized speed, termed a “half spin” either clockwise or counter-clockwise. Electrons adhere to Fermi-Dirac Statistics (ref [55]). Only one “fermion” is allowed in a given energy state with a specific spin. If an up (↑) spin is in the subshell, then a down (↓) spin can also be @ the same energy in the subshell. Subatomic fermions encountered in daily life, which have spin ½, include electrons, protons & neutrons.

Some particles that carry the forces of nature have integer spin & adhere to Bose-Einstein Statistics (ref [56]). Many “bosons” can exist @ the same state. Photons @ spin 1 carry the EM force & gravitons @ spin 2 carry the gravitational force (ref [57]). They are both bosons & travel @ light speed.

Therefore, as an atom’s spherical harmonic subshells are being filled by electrons, these quantum subshells get filled by a factor of two with an up-down (↑↓) spin electron pair.

A Completed Subshell: Electrons arrange themselves in shells & subshells around an atom’s nucleus to reduce energy (ref [58]). In each quantum shell, the electrons fill atomic subshells in a loose correspondence to spherical harmonics.

Noble Gas Electron Configurations in the Periodic Table							
sym	electron shell configuration					sym	subshells
₂ He	1s ²					-	1s ²
₁₀ Ne	1s ²	2s ²	2p ⁶			[₂ He]	2s ² 2p ⁶
₁₈ Ar	1s ²	2s ²	2p ⁶	3s ²	3p ⁶	[₁₀ Ne]	3s ² 3p ⁶
₃₆ Kr	1s ²	2s ²	2p ⁶	3s ²	3p ⁶	4s ²	3d ¹⁰ 4p ⁶
₅₄ Xe	1s ²	2s ²	2p ⁶	3s ²	3p ⁶	4s ²	3d ¹⁰ 4p ⁶ 5s ² 4d ¹⁰ 5p ⁶
row	1	2	3	4	5		

The electron configuration is described in the notation above. The right column contains an abbreviated designation. The s, p, d, f subshells have increasing angular momentum denoting ($l = 0, 1, 2, 3$). The (m) range ($(-l) \leq m \leq (+l)$) represent spatial orientations of the angular momentum subshell (l) for increasing states of orbital momentum. For up-down (↑↓) spin, electron pairs double up in every subshell.

With electrons, the noble gas electron states of Helium (₂He), Neon (₁₀Ne), Argon (₁₈Ar), Krypton (₃₆Kr) have extremely low energy stable states. Note that the 3d subshell is placed on row 4 of the periodic table, 1 row down. This subshell is filled after the 4s subshell instead of the 3p subshell on row 3. Electrons filling the 1s, 2s, 2p, 3s & 3p subshells shield the electrons in the 3d subshell from being filled in a spherical harmonic order within the atomic electron configuration.

Likewise, the 4f subshell is placed on row 6 of the periodic table, 2 rows down. The 4f subshell is filled after the 6s subshell instead of the 4d subshell of mathematical spherical harmonics.

Conductor or Not – Feel It: Many times, one can feel that a material is a “good” conductor. Metals feel colder than other material because they absorb or “conduct” heat readily. Copper ($_{29}\text{Cu}$), Silver ($_{47}\text{Ag}$) & Gold ($_{79}\text{Au}$) all have an s subshell incomplete after a full d subshell. This is a loosely bound single electron that is highly mobile to transport energy. In an electrical conductor, the metal absorbs one’s body heat when touched. The free electrons absorb heat from one’s hand & carry it elsewhere to distribute the energy throughout the conductor (ref [59]).

The elements $_{6}\text{C}$, $_{14}\text{Si}$, $_{32}\text{Ge}$ are called “semiconductors”. Coal, glass & concrete still feel cold to the touch, but not as much as a metal. For plastic, as on the insulation of a $_{29}\text{Cu}$ wire, the plastic feels warm. Almost all electrons are bound by covalent bonds in hydrocarbons; therefore the plastic appears warm to the touch & makes a good insulator for the contained electric current within the $_{29}\text{Cu}$ wire.

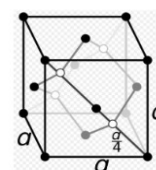
Element Groups Within Periodic Columns

Different elements arrange themselves into different categories based on their electron configurations. Elements in columns of the Periodic Table & listed below include the Coinage Metals (Group 11), the Boron Group (Group 13), the Carbon Group (Group 14) & the Pnictogen Group (Group 15).

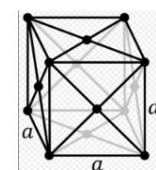
In many cases, elements within these groups are sufficient similarity so that application selections can be made down the column for cost & performance considerations in semiconductor applications. Reference [42] gives comprehensive numeric characteristics for the elements & used in the following tables. Electrical resistivity may vary with temperature & is given @ room temperature (20°C or 68°F) (ref [60]).

The Carbon Group: The $_{6}\text{C}$ atom in its diamond structure (ref [61]) forms the strongest crystal known, diamond due to covalent completion of its s & p subshells. As expected, the Mohs hardness of $_{6}\text{C}$ diamonds is @ the maximum measured. $_{14}\text{Si}$ & $_{32}\text{Ge}$ crystals of Group 14 decrease in melting temperatures, hardness & electrical resistivity due to active electrons further from their atomic nucleus. In these crystal forms, each atom forms 4 covalent bonds with 4 adjacent atoms (ref [62]).

Atomic Elements Comparison				
labels		Carbon Group (semi-conductors)		
Name		<u>Carbon</u>	<u>Silicon</u>	<u>Germanium</u>
atom# Symbol		₆ C	₁₄ Si	₃₂ Ge
electron configuration		[₂ He] 2s ² 2p ²	[₁₀ Ne] 3s ² 3p ²	[₁₈ Ar] 3d ¹⁰ 4s ² 4p ²
crystal (#atom)		diamond (8)	diamond (8)	face-centered (4)
hardness (Mohs)		10 (diamond)	6.5	6.0
melting pt. (°C)		3550 °C	1414 °C	938 °C
melting pt. (°F)		6422 °F	2577 °F	1720 °F
abundance (%/crust)		0.18%	27%	0.00014%
conductivity (W/m/K)		140 (graphite)	150	60
resistivity (nΩ · m)		10000	1000	500000



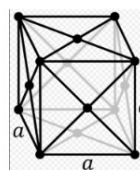
diamond



face-centered

The Coinage Metals: These elements of Group 11 have a face-centered cubic crystal structure (ref [63]). This atomic group is the most electrically conductive of all the elements (ref [64]). In each element, a single loosely bound electron is in the beginning of an outside s subshell. This “highly mobile” electron is available to redistribute energy changes whether from electrical fields or heat by redistributing itself. The Coinage Metals have moderate melting temperatures & Mohs hardness, below the crystal forms of the semiconductors. As expected, the conductivity values are high & electrical resistivity is orders of magnitude below that of the semiconductors. From percent abundance in earth’s crust, precious metals, ₄₇Ag & ₇₉Au, have microscopic occurrences; ₂₉Cu is the best choice for industry conductor applications.

Atomic Elements Comparison				
labels		Coinage Metals (metallic conductors)		
Name		<u>Copper</u>	<u>Silver</u>	<u>Gold</u>
atom# Symbol		₂₉ Cu	₄₇ Ag	₇₉ Au
electron configuration		[₁₈ Ar] 3d ¹⁰ 4s ¹	[₃₆ Kr] 4d ¹⁰ 5s ¹	[₅₄ Xe] 4f ¹⁴ 5d ¹⁰ 6s ¹
crystal (#atom)		face-centered (4)	face-centered (4)	face-centered (4)
hardness (Mohs)		3.0	2.5	2.5
melting pt. (°C)		1085 °C	962 °C	1064 °C
melting pt. (°F)		1985 °F	1764 °F	1947 °F
abundance (%/crust)		0.0068%	7.9e-6%	3.1e-7%
conductivity (W/m/K)		400	430	320
resistivity (nΩ · m)		17	16	22



face-centered

p-doped Elements: When solitary atoms of Group 13 are placed in a semiconductor crystal, the atoms provide three outer electrons to a 4-bond ₁₄Si lattice site. Because a covalent bond remains unfilled for the ₁₄Si lattice, a positively charged hole exists. In this atomic group, the melting point temperatures & crystal hardness decrease with increasing atomic number showing the weakness of active electrons further from their atomic nucleus. However, ₁₃Al has electrical resistivity & thermal conductivity only slightly higher than the Coinage Metals. Indeed, ₁₃Al is considered a conductor & used in high voltage power lines throughout the electrical power industry (ref [65]).

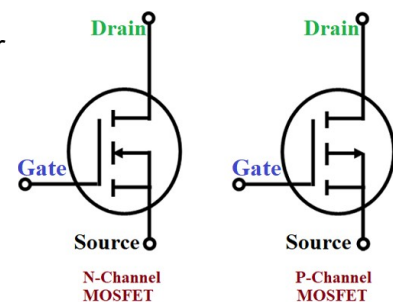
Atomic Elements Comparison			
labels	Boron Group (p-doped elements)		
Name	<u>Boron</u>	<u>Aluminum</u>	<u>Gallium</u>
atom# Symbol	₅ B	₁₃ Al	₃₁ Ga
electron configuration	[₂ He] 2s ² 2p ¹	[₁₀ Ne] 3s ² 3p ¹	[₁₈ Ar] 3d ¹⁰ 4s ² 4p ¹
hardness (Mohs)	9.3	2.8	1.5
melting pt. (°C)	2075 °C	660 °C	30 °C
(°F)	3767 °F	1220 °F	86 °F
abundance (%/crust)	0.00086%	8.1%	0.0019%
conductivity (W/m/K)	27	235	29
resistivity (nΩ · m)	10 ¹³	26	140

n-doped Elements: When solitary atoms of Group 15 are placed in a semiconductor crystal, the atoms provide 5 outer electrons to a 4-bond ₁₄Si lattice site. Because the bonds are filled for the ₁₄Si lattice with a free electron left over, a negative charge exists. Showing relatively weak crystal structures, this atomic group has low melting point temperatures & low hardness.

Atomic Elements Comparison			
labels	Pnictogen Group (n-doped elements)		
Name	<u>Phosphorus</u>	<u>Arsenic</u>	<u>Antimony</u>
atom# Symbol	₁₅ P	₃₃ As	₅₁ Sb
electron configuration	[₁₀ Ne] 3s ² 3p ³	[₁₈ Ar] 3d ¹⁰ 4s ² 4p ³	[₃₆ Kr] 4d ¹⁰ 5s ² 5p ³
hardness (Mohs)	~0	3.5	3.0
melting pt. (°C)	44 °C	817 °C	631 °C
(°F)	111 °F	1503 °F	1168 °F
abundance (%/crust)	0.099%	0.00021%	0.00002%
conductivity (W/m/K)	0.2	50.0	24.0
resistivity (nΩ · m)	100	300	400

The Hardware of One's & Zero's

An Exercise in Logic: Computers store data & number values in binary form using transistors (ref [17]). Likewise, computers perform math on binary numbers. Using transistors, the only math a computer can perform is binary math including addition, subtraction & multiplication. Division may involve an iterative process (ref [66]). The 1st three math evaluations are reduced to logic tables for binary input / output values using transistor schematics in chip design (ref [64]). Computers are “so smart” because of scale. When vacuum tubes were used in computers, the computer required an entire room (ref [67]). Now, that scale is miniaturized electronic elements by orders of magnitude, a laptop computer's scaled abilities increase by orders of magnitude.



Let's remind the reader, transistors were originally developed to replace vacuum tubes in amplifying weak EM signals. Dependent on “gate” voltage (V_G), if (V_G) is above a saturation voltage (V_S), the transistor will be in a specific binary state. If (V_G) is below a

cut-off voltage (V_C) then the transistor will be in the opposite binary state. The transistor state is based on current flow (open circuit) or no flow (closed circuit) between current “source” & “drain” (ref [68]).

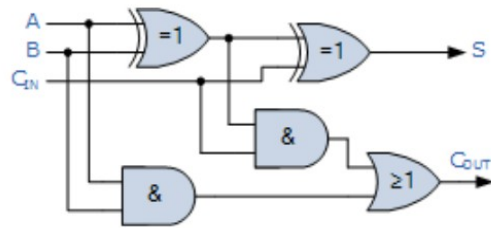
Example – Adding Two Bits:

Electrical Engineers implement a full binary adder with carry in the schematic to the right (ref [69]).

To add two non-negative integer values stored in two 32-bit memory addresses, the values are 1st loaded from random

access memory (RAM) into special registers & the equivalent significant bits of each value are lined up for analysis. The special registers account for a carry (C_{IN}) bit from the lower significant bit addition, then output the result in a Sum (S) bit with an evaluated carry (C_{OUT}) bit.

Logic Table				
Binary Add With Carry				
Input			Output	
C_{IN}	B	A	S	C_{OUT}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1



The process involves taking in 3 binary values stored as 3 bits, consulting two separate logic tables that considers all possible ($2^3 = 8$) input combinations, then setting the two output bits based on this hardwired analysis. In the logic table above, the bits to add are in registers (A) & (B) with a carry value @ (C_{IN}) (ref [69]). The logic table evaluates the required bit (S) & (C_{OUT}) carry values from the 3 input bits. If the logic table is applied 32 times between two 32-bit values from least to most significant bits, then the computer has **correctly** added two non-negative integer values via logic tables. For a computer, the complicated is reduced to the very simple & repeated billions of times over!

AND Gate	NAND Gate	OR Gate	XOR Gate

The above schematics & logic tables were found in refs [70] & [71] & [72]. This adder design breaks the evaluation down into three fundamental types of “logic gates”: 1) XOR gates to evaluate the Sum bit (S); 2) AND & OR gates to evaluate the carry bit (C_{OUT}). The XOR gate is grouped into a combination of simpler AND, NAND & OR gates. In schematics, these gates have the above notations, logic tables & transistor

circuit implementations. The XOR gate schematic is found in ref [73].

Electronic Engineering, when applying computer technology, must accurately apply AND, NAND, OR, XOR gates & logic tables which are quite far removed from the fundamental equations of classical EM, i.e., Maxwell's Equations ([eduDipole.pdf](#)). In applying spherical harmonics of the Periodic Table, Quantum Theory effects are prevalent & Maxwell's Equations of the 19th century are only partially valid. However, this is the essence of Engineering which must interface with all aspects of Science, Mathematics & Statistics (when required) to create a product which improves the human condition.

Higher Computer Math: The "Do Loop" alluded to for computer division implies an "iterative process" which is the secret to most higher math functions (refs [74] & [75]), including trigonometric, logarithmic & exponential functions. Look-up tables can be employed whereby a curve is stored by incremental data points & an evaluation is achieved through interpolation. However, industrious mathematicians through the ages have derived infinite sums & infinite multiplication products for many of the above functions. When these formulas are successively evaluated, the formulas converge on the correct answer to a desired accuracy. Of course, these "magic" algorithms still implement nothing more than the computer's grade school abilities of addition, subtraction, multiplication & division.

Improved Computers: Computers are essential in designing computer chips! Computer-Aided-Design (CAD) software is used extensively to layout the repetitive elements of the microscopic chip circuits. Which came 1st, the CAD chip design workstation or the computer chip?

Forty year ago, I was impressed with my 5 MHz 16-bit Intel 8086 microprocessor & its 8087 math co-processor for floating point calculations. I had to purchase the co-processor separately (ref [76]). Now, microprocessors have a thousand times the clock rates or greater, a 64-bit data word can be transferred in parallel in total per cycle, not just 16-bits, the math co-processor is integrated into the microprocessor & there may be 7 or 8 separate microprocessor cores multi-processing on one chip.

While the fundamentals of a microprocessor have remained about the same since the 1980's, everything has increased in scale by orders of magnitude. As an example, the amount of RAM (random access memory) available to a processor is not part of the SI metric system, however, its magnitude range is expressed in bytes approximated from their naming convention ([siData.pdf](#)). By industry standard, binary quantities are expressed in "kibibytes" (1 KibiByte = 1 KiB = 2^{10} bytes = 1024 bytes with 8 bits in a byte, ref [77]). For layman consumers, the middle "i" is omitted & 1 KiB \approx 1 KB (1 KiB has 2.4% more RAM than 1 KB of RAM). Decades ago, laptops had kilobytes (KB) of RAM, then megabytes (MB) of RAM, then gigabytes (GB) of RAM, & now a terabyte (TB) of RAM can be purchased for a laptop.

Making a Semiconductor Chip

The microchip starts as a flat pure crystal ${}_{14}\text{Si}$ wafer with either n-doped or p-doped

chemicals already included. On that, is built successive layers of electronic elements such as transistors, capacitors, parallel metallic conductor pathways & other elements to support the electronics of the chip.

In 1796, actor & playwright, Alois Senefelder (1771 – 1834) invented a way to transfer graphics on to paper for printed music & journalism. Fortunately, Mr. Senefelder is remembered far more for his lithography process than his acting & play scripts. His method printed graphics by “etching” the image in stone, putting ink on the stone, then transferring the image to paper (ref [78]).

Chip Fabrication: The $_{14}\text{Si}$ semiconductors “fabrication process is performed in highly specialized semiconductor fabrication plants, also called foundries or ‘fabs’ “ (ref [79]). The process is entirely automated in “clean rooms” that have a 100% nitrogen (N_2) atmosphere to reduce imperfect chip fabrication. N_2 is an inert, odorless, colorless diatomic gas which composes of about 78% of the air we breath (ref [42]). However, a 100% N_2 environment is lethal because no oxygen is present.

Microchip manufacturers carry Mr. Senefelder’s idea further as photo-lithography, functional electronics are produced by etching silicon. Just as a drive-in movie projector can illuminate a two-story screen with a Hollywood movie, the CAD software used to design the chip, calculates electronic images & projects them on to the microscopic chip area with camera lens @ a very small scale.

For example, to lay some $_{29}\text{Cu}$ pathways on a chip, manufacturers coat a microchip completely with metallic $_{29}\text{Cu}$ through vapor deposition. Then, etching chemicals are applied that are sensitive to light. CAD generated templates are projected on the coated chips to take away material where the metallic coating is not required for electronic functions.

Chip “fabrication can take up to 15 weeks, with 11–13 weeks being the industry average” (ref [79]). Along with vapor deposition of chemicals, ion doping, chemical cleaning, thermal oxidation & other processes occur before chip completion. Several chips can be produced on a single wafer @ a time, then sliced apart. For quality assurance, electrodes are attached to the finished product & each electronic element within each chip is tested for proper functional requirements. A quarter of the chips may be rejected because of fabrication defects (ref [80]).

Conclusion

Computer chips have been a great success story of Electrical Engineering. In addition, what has been eliminated is the room of professional mathematicians “reckoning” away @ long-hand calculations of Engineering or Science for some societal issue.

One of the 1st programming languages invented was FORTRAN which is an acronym for FORmula TRANslation (ref [81]). When I worked in a civil service capacity for the US military, we supported “legacy code” written in FORTRAN version 66, an ANSI standard codified in 1966. The program was originally written on punch cards with **no** comment statements. Hell, the more comment statements one included in the punch card stack, the heavier & more unwieldy the computer program storage was. The “legacy code” has

been lifted from its punch cards, long ago. In addition, the legacy code has performed in its military role well & never encountered a problem. The logic implementing the code was sound. "If it works, Don't fix it!"

In our **Brave New World**, as we put full computers in our refrigerators to help us build our grocery lists, let's not forget the technological successes that have been achieved to make the ubiquitous computer chip possible.

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